American University of Beirut
Math 204
Quiz I (Fall 2014)

Time 75 minutes .

Name: _____ **ID#: _____**

Circle your problem solving section number below:

- **Instructor: <u>Ms Joumana Tannous</u>**

Section 1 @ 1:00 M **Section 2 @ 3:00 M** **Section 3 @ 4:00 M**

- **Instructor: <u>Mrs Maha Itani-Hatab</u>**

Section 4 @ 11:00 Tu **Section 5 @ 8:00 Tu** **Section 6 @ 12:30 Tu**

- **Instructor: <u>Ms. Michella Bou Eid</u>**

Section 7 @ 12:30 Th **Section 8 @ 2:00 Th** **Section 9 @ 5:00 Th**

- **Instructor: <u>Ms Najwa Fuleihan</u>**

Section 10 @ 8:00 Tu **Section 11 @ 12:30 Tu** **Section 12 @ 11:00 Tu**

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1. Construct the 3×3 lower triangular matrix A whose non zero entries a_{ij} are

(8 pts) $$a_{ij} = \begin{cases} 2^j - i & \text{if} \quad i \neq j \\ i^2 - 2 & \text{if} \quad i = j \end{cases}$$

2. Evaluate $$\begin{bmatrix} 2 & 1 \end{bmatrix}\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & -2 \end{bmatrix} + 2\begin{bmatrix} -2 & 1 \\ 0 & -1 \\ 2 & 4 \end{bmatrix}^T \right)$$

(8 pts)

3. Find x and y so that the operations can be performed

$$\begin{pmatrix} -3 & x \\ 2y & 0 \end{pmatrix}^T + \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 5 & -4 \end{pmatrix}$$

(8 pts)

4. Let A be a (2 × 3) matrix, B a (2× 2) matrix, C a (2× 3) and D a (3 × 3) matrix.

a. Determine, if possible, the size (dimension) of each of the following matrices

(8 pts)

- $\left(A^T B^2 C\right)^0$
- $\left(BAD\right)^0$
- $B\,C^{-1}$
- $\left(A+C\right)D$

b. Determine, if possible, the size (dimension) of the identity matrix I and the zero matrix O so that the operations can be performed

$$AD^{-1}I - B\,O$$

(4 pts)

5. Given the matrix $A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 6 & -3 \\ 2 & -4 & 0 \end{pmatrix}$.

a) Rewrite the first two columns (Use the method of repeated columns) to calculate the determinant of A.

(3 pts)

b) Consider the system $\begin{cases} 3x + 2y - z = 1 \\ x + 6y - 3z = 2 \\ 2x - 4y = 0 \end{cases}$ .Use Cramer's rule to find **<u>only</u>** y.

(5 pts)

6. Explain why the value of the following determinant $\begin{vmatrix} 2 & 3 & 4 & 5 \\ 4 & 0 & 8 & 1 \\ 2 & 3 & 4 & 5 \\ 7 & 6 & 0 & 9 \end{vmatrix}$ is zero.

(3 pts)

7. Consider the matrix $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix}$. Find A^{-1} using the Gaussian elimination.

(16 pts)

<u>Then use it to solve the system</u>: $\begin{cases} x - 3y + 5z = 5 - x \\ 3x - y - z = 1 \\ x + 2y - z = 2y \end{cases}$

8. Let $A = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 6 \\ 1 & -1 & 4 \\ 0 & 1 & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 0 & 0 \\ 11 & 1 & 0 \\ 5 & 8 & 1 \end{pmatrix}$.

a. Use the method of cofactors to evaluate the determinant of the matrix B.

(4 pts)

b. Let I be the identity matrix and D a 3×3 matrix. Determine the value of the determinant of the matrix D so that $\det\left(I^{-1} A\right) = \det\left(2\, D\, C^{-1} B^{T}\right)$.

(8 pts)

c. Find $\det\,(2A)^{-1}$

(4 pts)

9. Solve the matrix equation

$$\begin{pmatrix} 3y+1 \\ x-5 \\ xy-2 \end{pmatrix} + 2\begin{pmatrix} y-2 \\ x^2+x \\ y+3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3x+3 \\ x^2y \end{pmatrix}$$

(8 pts)

10. Find a 2×2 matrix A such that $\left((2A)^T - I\right)^{-1} = \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$

(8 pts)

11. Let A be a 2×2 matrix such that $\det(A^4) + \det(AA^T) = 0$. Show that A has no inverse.

(5 pts)